Bayesian Networks in Reliability: Some Recent Developments

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Extended abstract

A Bayesian Network (BN), see (Pearl 1988; Jensen 1996; Cowell et al. 1999; Jensen 2001), is a compact representation of a multivariate statistical distribution function. A BN encodes the probability density function governing a set of random variables $\{X_1, \ldots, X_n\}$ by specifying a set of conditional independence statements together with a set of conditional probability functions. More specifically, a BN consists of a qualitative part, a *directed acyclic graph* where the nodes mirror the random variables X_i, and a quantitative part, the set of conditional probability functions. An example of a BN over the variables $\{X_1, \ldots, X_5\}$ is shown in Figure 1, only the qualitative part is given. We call the nodes with outgoing edges pointing into a specific node the *parents* of that node, and say that X_j is a *descendant* of X_i if and only if there exists a directed path from X_i to X_j in the graph. In Figure 1 X_1 and X_2 are the parents of X_3, written $\mathrm{pa}(X_3) = \{X_1, X_2\}$ for short. Furthermore, $\mathrm{pa}(X_4) = \{X_3\}$ and since there are no directed path from X_4 to any of the other nodes, the descendants of X_4 is the empty set.

Now, the edges of the graph represents the assertion that a variable is conditionally independent of its non-descendants in the graph given its parents in the same graph. (Other conditional independence statements can be read off the graph by using the rules of *d-separation*, (Pearl 1988).) The graph in Figure 1 does for instance assert that X_4 is conditionally independent of $\{X_1, X_2, X_5\}$ when conditioned on X_3.

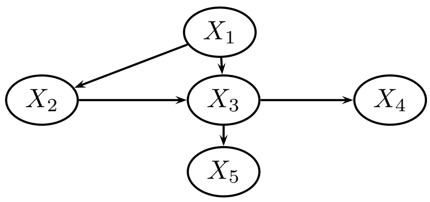

Figure 1: An example BN over the nodes $\{X_1, \ldots, X_5\}$. Only the qualitative part of the BN is shown.

When it comes to the quantitative part, each variable is described by the conditional probability function of that variable *given the parents* in the graph, i.e., the collection of conditional probability functions $\{f(x_i \mid \mathrm{pa}(x_i))\}_{i=1}^{n}$. The underlying assumptions of conditional independence encoded in the graph allow us to calculate the joint probability function as

$$f(x_1, , \ldots, x_n) = \prod_{i=1}^{n} f(x_i \mid \mathrm{pa}(x_i)), \tag{1}$$

see, e.g., (Pearl 1988; Jensen 1996). Note that a BN can represent *any* probability density function, and through its factorized representation (Eqn. 1), it does so in a cost-efficient way. Furthermore, efficient algorithms for calculating arbitrary marginal distributions, say, $f(x_i, x_j, x_k)$ as well as conditional distributions, say, $f(x_i, x_j \mid x_k, x_\ell)$, make BNs well suited for modelling complex systems.

The history of BNs in reliability can (at least) be traced back to Barlow (1988). More recently, BNs have found applications in, e.g., software reliability (Fenton et al. 1998; Gran 2002), fault finding systems (Heckerman et al. 1995; Jensen et al. 2001), maintenance modelling (Langseth and Lindqvist 2003), and general reliability modelling (Bobbio et al. 2001; Langseth 2002; Ingleby and West 2003).

This talk will introduce the basic properties of Bayesian network models, and then describe some recent developments that make BN models well suited for applications in reliability.

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