The Use of MLE in Sequential Chi-square Tests for Embedded Samples

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ABSTRACT

If observations are more expensive (or it is difficult to get them, for example, in reliability, in insurance and in medicine), then to test null hypothesis can be useful some tests, by which for receiving the hypothesis can be used by not large number observations, but to reject it by samples of large size. For this purpose Zakharov, Sarmanov and Sevastyanov (1969), hereafter referred to as ZSS, suggested chi-square type sequential test based on the vector statistics

$$X_n^2 = \left(X_{n_1}^2, X_{n_2}^2, \ldots, X_{n_m}^2 \right)^T, \tag{1}$$

where $n_1 < n_2 <\ldots< n_m$ - increasing sizes of embedded one to another samples, $X_{n_i}^2$ - Pearson's standard statistics. ZSS (1969) obtained the limiting Laplace transformation of statistics (1) under simple null hypothesis and convergence alternatives. Joint distribution of components of statistic X_n^2 was established Jensen (1974).

It is actually to explore the large-sample behavior of statistics

$$X_n^2(\theta,\varphi) = \left(X_{n_1}^2(\theta,\varphi), X_{n_2}^2(\theta,\varphi), \ldots, X_{n_m}^2(\theta,\varphi) \right)^T \tag{2}$$

for composite null hypothesis $H_0 : \eta = \eta_0$ against the sequence of convergence hypothesis $H_{1n} : \eta_n = \eta_0 + n^{-1/2}\gamma_n$, where $\gamma_n \to \gamma$ for fixed $\gamma \in R^p$ as $n_1 \to \infty$.

There are three practically interesting moments of this consideration. The first – in all components of statistics (2) θ replaced to θ_{n_1}, calculated by the first sample size n_1, secondly – in each components $X_{n_i}^2$ θ replaced to θ_{n_i}, calculated by sample size n_i $(i = 1,\ldots, m)$ and the third – as the first case θ replaced to θ_{n_m}, calculated by the last sample size n_m. Formanov and Mirvaliev (2000) studied case two for sequential generalized χ^2 type statistics (see also Moore and Spruill (1975)) and wide class of estimates. Mirvaliev (2002) consider the first and the third cases for multinomial MLE (or asymptotically equivalent estimates).

A Laplacian analysis of the distribution of sequential chi-square statistics for embedded samples is developed in this paper. There are found the limiting Laplace transformation of vector statistics (2) when the unknown parameter is replaced to MLE by the first and the last samples.

References

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