Investigating doubled aircraft inspection frequency strategy for exponential fatigue crack growth model

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Abstract

Switching to double inspection frequency after the first fatigue crack discovery during aircraft inspection in operation is investigated. For inspection program development on a base of lifetime approval test result processing the use of p-set function and minimax approach is offered.

1. Introduction

Inspection program development should be made on the base of processing of approval lifetime test result, when we should make some redesign of the tested system if any requirement is not met. Here we consider some example of p-set function application to the problem of development and control of inspection program. We make assumption that some Structural Significant Item (SSI), the failure of which is the failure of the whole system, is characterized by a random vector (r.v.) ($\boldsymbol{T_d}$, $\boldsymbol{T_c}$), where $\boldsymbol{T_c}$ is critical lifetime (up to failure), $\boldsymbol{T_d}$ is service time, when some damage (fatigue crack) can be detected. So we have some time interval, such that if in this interval some inspection will be fulfilled, then we can eliminate the failure of the SSI. We suppose also that a required operational life of the system is limited by so-called Specified Life (SL), $\boldsymbol{t}_{SL}$, when system is discarded from service. P-set function for random vector is a special statistical decision function, which is defined in following way.

*Let **Z** and **X** are random vectors of **m** and **n** dimensions and we suppose that it is known the class* $\{\boldsymbol{P_\theta}, \theta \in \boldsymbol{\Omega}\}$ *to which the probability distribution of the random vector* **W=(Z,X)** *is assumed to belong. The only thing we assume to be known about the parameter* θ *is that it lies in a certain set* $\boldsymbol{\Omega}$, *the parameter space. If* $\boldsymbol{S_Z}(\boldsymbol{x}) = \bigcup_{\boldsymbol{i}} \boldsymbol{S}_{\boldsymbol{Z},\boldsymbol{i}}(\boldsymbol{x})$ *is such set of disjoint sets of **z** values as function of **x** that* $\sup_\theta \sum_{\boldsymbol{i}} \boldsymbol{P}(\boldsymbol{Z} \in \boldsymbol{S}_{\boldsymbol{Z},\boldsymbol{i}}(\boldsymbol{x})) \le \boldsymbol{p}$ *then statistical decision function* $\boldsymbol{S_z(x)}$ *is p-set function for r.v.* **Z** *on the base of a sample* $\boldsymbol{x=(x_1,...,x_n)}$.

Later on the value $\boldsymbol{x}$, observation of the vector $\boldsymbol{X}$, would be interpreted as estimate $\hat{\theta}$ of parameter θ, $\boldsymbol{Z}$ would be interpreted as some random vector-characteristic of some SSI in service: $\boldsymbol{Z} = (\boldsymbol{T_d}, \boldsymbol{T_c})$.

2. Inspection program development

By processing results of some special approval test (full-scale fatigue test of airframe, for instance), we can get estimate $\hat{\theta}$ of parameter θ. The problem is to find (in general case) a vector function $t(\hat{\theta})$, where $\boldsymbol{t} = (\boldsymbol{t_1}, \boldsymbol{t_2}, ..., \boldsymbol{t_n})$, $\boldsymbol{t_i}$ is time moment of i-th inspection, ***i=1,2,…,n, n*** is inspection number, $\boldsymbol{t}_{n+1} = \boldsymbol{t}_{SL}$, in such a way, that failure probability of SSI under consideration does not exceed some small value ε:

$$\sup_{\theta} \boldsymbol{p}_f(\theta, \boldsymbol{t}) \le \varepsilon,$$

where $\boldsymbol{p}_f(\theta, \boldsymbol{t}) = \Sigma^r{}_{i=1} \boldsymbol{P}(\boldsymbol{T}_{i-1} \le \boldsymbol{T}_d < \boldsymbol{T}_c < \boldsymbol{T}_i)$, $\boldsymbol{T}_1, \ldots, \boldsymbol{T}_n$ are moments of inspections: r.v. $\boldsymbol{T} = (\boldsymbol{T}_1, \ldots, \boldsymbol{T}_n) = t(\hat{\theta})$; $\boldsymbol{T}_0$=0; $\boldsymbol{T}_{n+1} = \boldsymbol{t}_{SL}$. This means that vector function $t(\hat{\theta})$ in fact defines some $\boldsymbol{p}$-set function for vector $(\boldsymbol{T}_d, \boldsymbol{T}_c)$ at $\boldsymbol{p}=\boldsymbol{\varepsilon}$.

For simplicity purpose let all inspection intervals are equal. Now probability of failure will be function of θ and $\boldsymbol{n}$ and we'll denote it by $\boldsymbol{p}_f(\theta, \boldsymbol{n})$. We suppose that $\boldsymbol{p}_f(\theta, \boldsymbol{n})$ monotonically decreases when n increases and $\lim_{n \to \infty} \boldsymbol{p}_f(\theta, \boldsymbol{n}) = \boldsymbol{0}$ for all θ. Let $\boldsymbol{n}(\theta, \varepsilon)$ is minimal inspection number $\boldsymbol{n}$ at which $\boldsymbol{p}_f(\theta, \boldsymbol{n}) \le \varepsilon$, where ε is some small value. But true value of θ is not known. So $\hat{\boldsymbol{n}} = \boldsymbol{n}(\hat{\theta}, \varepsilon)$ and $\hat{\boldsymbol{p}}_f = \boldsymbol{p}_f(\theta, \hat{\boldsymbol{n}})$ are random variables. We suppose, that we begin the commercial production and operation **only** if some specific requirements to reliability are met. Let us denote in general case this event as $\hat{\theta} \in \Theta_0$, where $\Theta_0 \subset \Omega$ is some part of parameter space. We suppose, that if $\hat{\theta} \notin \Theta_0$ (estimate of required inspection number for some fixed ε exceeds some threshold $\boldsymbol{n}_{\max}$ or estimate of expectation value of $\boldsymbol{T}_c$ is too small in comparison with $\boldsymbol{t}_{SL}$), then we make redesign of the SSI in such a way, that probability of failure after this redesign will be equal to zero. Let us define

$$\hat{\boldsymbol{p}}_{f0} = \begin{cases} \boldsymbol{p}_f(\theta, \hat{\boldsymbol{n}}) \; \boldsymbol{if} \;\; \hat{\theta} \in \Theta_{\boldsymbol{0}}, \\ \boldsymbol{0} \;\; \boldsymbol{if} \;\; \hat{\theta} \notin \Theta_{\boldsymbol{0}}. \end{cases}$$

For this type of strategy the mean probability of fatigue failure $\boldsymbol{w}(\theta, \varepsilon) = \boldsymbol{E}_\theta(\hat{\boldsymbol{p}}_{f0})$ is a function of θ and ε. If for limited $\boldsymbol{t}_{SL}$ it has a maximum, depending on ε then the choice of maximal value of $\varepsilon = \varepsilon^*$ for which $\max_{\theta} \boldsymbol{w}(\theta, \varepsilon^*) \le \boldsymbol{1} - \boldsymbol{R}$ and inspection number $\boldsymbol{n} = \boldsymbol{n}(\hat{\theta}, \varepsilon^*)$ is such strategy for which required reliability $\boldsymbol{R}$ is provided.

3. Example

The simplest example of considered approach with unchangeable interval between inspections is given in referenced papers. The disadvantage of this strategy is a large number of inspections in the initial period when the probability to discover the fatigue crack is negligibly small. In this paper we consider more complex strategy when the inspection program, preliminary developed on the base on approval test information, later on can be changed after discovery of first fatigue crack.The numerical calculations will be based on exponential model of fatigue crack growth when the crack size $\boldsymbol{a}(\boldsymbol{t}) = \boldsymbol{a}(\boldsymbol{0}) \exp(\boldsymbol{Qt})$.

Then $\boldsymbol{T}_d = (\log \boldsymbol{a}_d - \log \boldsymbol{a}_0)/\boldsymbol{Q} = \boldsymbol{C}_d/\boldsymbol{Q}$, $\quad \boldsymbol{T}_c = (\log \boldsymbol{a}_c - \log \boldsymbol{a}_0)/\boldsymbol{Q} = \boldsymbol{C}_c/\boldsymbol{Q}$,

where $\boldsymbol{a_0}$ is $\boldsymbol{a(0)}$; $\boldsymbol{a}_d$, $\boldsymbol{a}_c$ are crack sizes corresponding to $\boldsymbol{T}_d$ and $\boldsymbol{T}_c$. In the simplest case let us suppose that a_0, a_d, a_c, $\boldsymbol{C}_d$ and $\boldsymbol{C}_c$ are constants. Usually it is assumed that random variable $\log(\boldsymbol{T}_C) = \log(\boldsymbol{C}_C) - \log(\boldsymbol{Q})$ has normal distribution. So **log(*Q*)** has normal distribution also, which we denote by $N(\theta_0,\ \theta_1^{\ 2})$. Suppose in operation there is a park of N aircraft of the same type. And we choose the following strategy. We develop two inspection programs with n and 2n inspections. We begin the operation of the park, using first program. But after at least one crack discovery or in case of fatigue failure of at least one aircraft we two times decrease interval between inspections on remain park of (N-1) aircraft. For example, in Fig. 1 the graph is shown corresponding for initial number of inspections n=2. After change of frequency (CF) the remaining time intervals are splits into two parts.

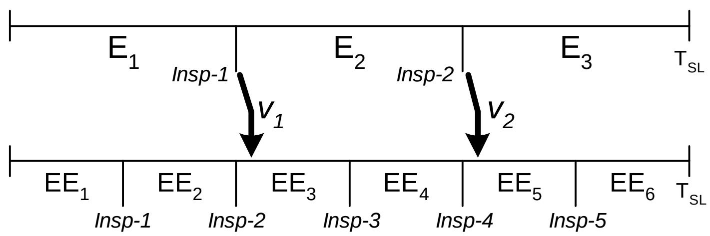

Figure 1: Switching to double inspection frequency from initially 2-inspections model

We suppose that after CF we continue the operation of every aircraft up to specified life but this time independently one from another and after retrofit of aircraft on which the fatigue crack was discovered the probability of its failure up to specified life will be equal to zero. Let us refer to this strategy as SWn-strategy. For failure probability calculation we need to use some results of Markov Chains theory. We define the set of states in following way. Let us denote the service of aircraft in certain ith interval $(t_{i-1,}t_i)$ as a state E_i. For all $i \leq (n+1)$ there are three possible transitions from this state to another states, which are represent (1) transition into next $(i+1)^{th}$ time interval or, if i=n+1, successful end of service (absorbing state E_{n+2}), (2) transition into absorbing E_{n+3} state, corresponding to the fatigue failure, and (3) transition into absorbing E_{n+4} state, corresponding to discovery of the fatigue crack. The corresponding probabilities we'll denote by $\boldsymbol{u_i}$, $\boldsymbol{q_i}$, $\boldsymbol{v_i}$. For absorbing states there are units in main diagonal. All the others probabilities of the considered matrix of transition probabilities are equal to 0. Then conditional probabilities $\boldsymbol{u_i}, \boldsymbol{q_i}$ are defined by formulas

$$\boldsymbol{u_i} = \Phi\left(\left(\ln(\boldsymbol{C_d} / \boldsymbol{t_i}) - \theta_0\right)/\theta_1 \right)/\Phi\left(\left(\ln(\boldsymbol{C_d} / \boldsymbol{t_{i-1}}) - \theta_0\right)/\theta_1 \right),$$

$$\boldsymbol{q_i} = \max(\boldsymbol{0}, (\Phi\left(\left(\ln(\boldsymbol{C_d} / \boldsymbol{t_{i-1}}) - \theta_0\right)/\theta_1\right) - \Phi\left(\left(\ln(\boldsymbol{C_c} / \boldsymbol{t_i}) - \theta_0\right)/\theta_1\right))/\Phi\left(\left(-\ln(\boldsymbol{C_d} / \boldsymbol{t_{i-1}}) + \theta_0\right)/\theta_1\right)),$$

where $\Phi(.)$ is distribution function of standard normal variable. It is clear that $\boldsymbol{v_i} = \boldsymbol{1} - \boldsymbol{u_i} - \boldsymbol{q_i}$ .It is clear also, that if we consider a park of N aircraft of the same type and if we are interested to know the probabilities of the failure of at least one aircraft or crack discovery in at least one aircraft of the park then instead of q_i and u_i we should use $\boldsymbol{q_{i,N}} = \boldsymbol{1} - (\boldsymbol{1} - \boldsymbol{q_i})^{\boldsymbol{N}}$ and $\boldsymbol{u_{i,N}} = (\boldsymbol{u_i})^{\boldsymbol{N}}$.

In general case by the use of relevant formulas for absorbing Markov Chains and Monte Carlo method for modelling $\hat{\theta}_{\boldsymbol{0}}$ we can calculate the function $\boldsymbol{w}(\theta,\varepsilon) = \boldsymbol{E}_\theta(\hat{\boldsymbol{p}}_{\boldsymbol{f0}})$, the average probability of failure of one aircraft in the park of N aircraft for SWn –strategy. Example of this calculation, the probability of redesign and corresponding initial data are shown in Fig.2. Let us explain that in considered example we have event $\hat{\theta} \notin \Theta_0$ if (1) for ε_1=0.0001 (in this case approximate value of probability of at least one failure in park $\varepsilon = \varepsilon_1$N=0.01) the required of inspections $\hat{\boldsymbol{n}} = \boldsymbol{n}(\hat{\theta},\varepsilon)$ is more than 3 or (2) estimate of mean $\boldsymbol{T_c}$ lesser than $\boldsymbol{t_{SL}}$. The maximum of the function $\boldsymbol{w}(\theta,\varepsilon) = \boldsymbol{E}_\theta(\hat{\boldsymbol{p}}_{\boldsymbol{f0}})$ for $\varepsilon = \varepsilon_1$N=0.01 is equal to 0.0015 .

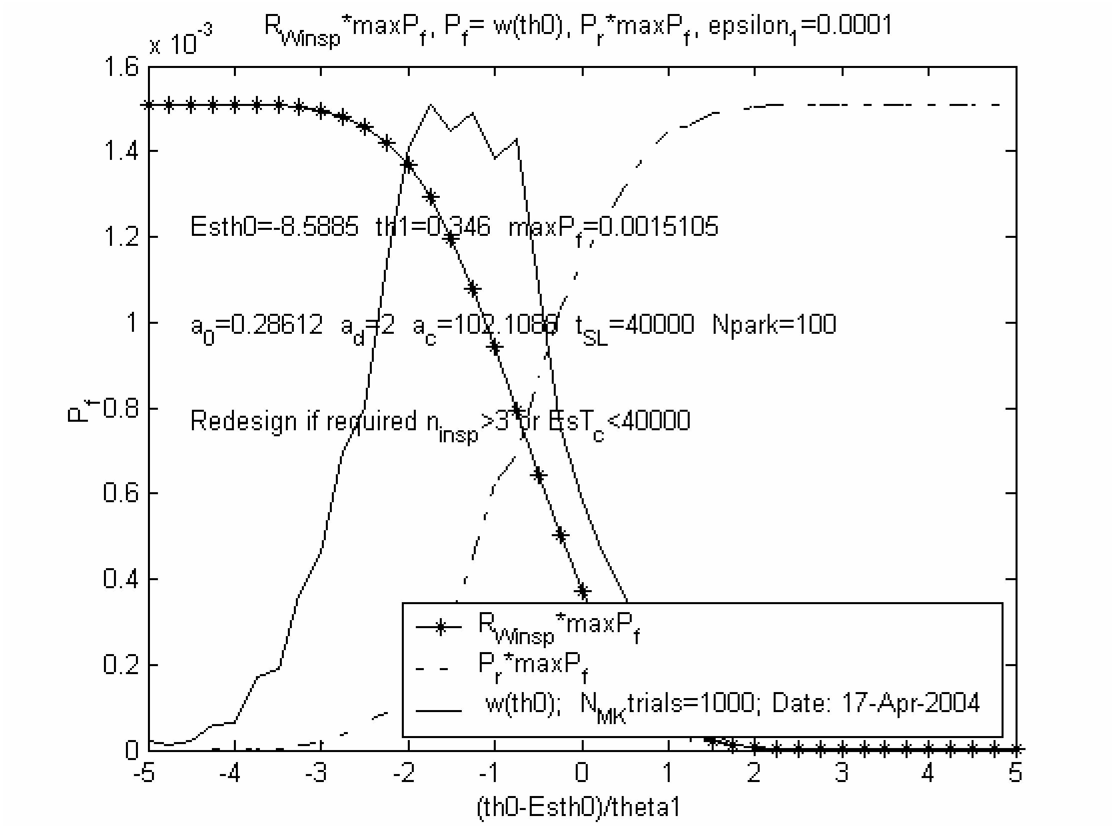

Figure 2: Function $\boldsymbol{w}(\theta,\varepsilon) = \boldsymbol{E}_\theta(\hat{\boldsymbol{p}}_{\boldsymbol{f0}})$ for ε_1=0.0001, Esth0=$\hat{\theta}_{\boldsymbol{0}}$, EsTc is estimate of $\boldsymbol{E}(\boldsymbol{T_c})$, N_{MK} trials is number of Monte Carlo trials (number of $\hat{\theta}_{\boldsymbol{0}}$ for every $\theta_{\boldsymbol{0}}$), P_r is redesign probability, R_{Winsp} is reliability without inspections.

So if required reliability (of one aircraft) is equal to 0.9985 then we for the SWn-strategy we should choose the inspection number for $\varepsilon = \boldsymbol{N}\varepsilon_1$=0.01. For the considered example $\hat{\boldsymbol{n}} = \boldsymbol{n}(\hat{\theta},\varepsilon\ )$=2.

References

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