Sensitivity Analysis in Conjunction with Evidence Theory Representations of Uncertainty

J.C. Helton,[1] W.L. Oberkampf, J.D. Johnson
Sandia National Laboratories, Albuquerque, NM 87185-0779

Uncertainty analysis and sensitivity analysis should be important components of any analysis of a complex system, with (i) uncertainty analysis providing a representation of the uncertainty present in the estimates of analysis outcomes and (ii) sensitivity analysis identifying the contributions of individual analysis inputs to the uncertainty in analysis outcomes.[1] Probability theory provides the mathematical structure traditionally used in the representation of epistemic (i.e., state of knowledge) uncertainty, with the uncertainty in analysis outcomes typically represented with probability distributions and summarized as cumulative distribution functions (CDFs) or complementary cumulative distribution functions (CCDFs).[2-4] A variety of sensitivity analysis procedures have been developed for use in conjunction with probabilistic representations of uncertainty, including differential analysis,[5, 6] the Fourier amplitude sensitivity test (FAST) and related variance decomposition procedures,[7-11] regression-based techniques,[12, 13] and searches for nonrandom patterns.[14]

Although probabilistic representations of uncertainty have been successfully employed in many analyses, such representations have been criticized for inducing an appearance of more refined knowledge with respect to the existing uncertainty than is really present.[15, 16] Much of this criticism derives from the use of uniform distributions to characterize uncertainty in the presence of little or no knowledge with respect to where the appropriate value to use for a parameter is located within a set of possible values. As a result, a number of alternative mathematical structures for the representation of epistemic uncertainty have been proposed, including evidence theory, possibility theory, and fuzzy set theory.[17]

Evidence theory provides a promising alternative to probability theory that allows for a fuller representation of the implications uncertainty than is the case in a probabilistic representation of uncertainty. In particular, evidence theory involves two representations of the uncertainty associated with a set of possible analysis inputs or results: (i) a belief, which provides a measure of the extent to which the available information implies that the true value is contained in the set under consideration, and (ii) a plausibility, which provides a measure of the extent to which the available information implies that the true value might be contained in the set under consideration. One interpretation of the belief and plausibility associated with a set is that (i) the belief is the smallest possible probability for the set that is consistent with all available information and (ii) the plausibility is the largest possible probability for the set that is consistent with all available information. An alternative interpretation is that evidence theory is an internally consistent mathematical structure for the representation of uncertainty without any explicit conceptual link to probability theory. The mathematical operations associated with evidence theory are the same for both interpretations. Just as probability theory uses CDFs and CCDFs to summarize uncertainty, evidence theory uses cumulative belief functions (CBFs), cumulative plausibility functions (CPFs), complementary cumulative belief functions (CCBFs), and complementary cumulative plausibility functions (CCPFs) to summarize uncertainty.

Although evidence theory is beginning to be used in the representation of uncertainty in applied analyses, the authors are unaware of any attempts to develop sensitivity analysis procedures for use in conjunction with evidence theory. Due to the importance of sensitivity analysis in any decision-aiding analysis, the potential usefulness of evidence theory will be enhanced if meaningful and practicable sensitivity analysis procedures are available for use in analyses that employ evidence theory in the representation of uncertainty. As a result, the focus of this presentation is on the development of sensitivity analysis procedures for use in conjunction with evidence theory representations of uncertainty.

The primary emphasis is on sensitivity analysis procedures based on complete variance decompositions. [8-11] The underlying idea comes from viewing CBFs and CPFs (or, equivalently, CCBFs and CCPFs) as defining envelopes that contain all possible distributions for a given analysis result that are consistent with available information on the uncertain analysis inputs. Each of the possible distributions contained in such an envelope derives from possible distributions for the uncertain analysis inputs and thus has its own variance decomposition. The end result is that there is not a unique variance decomposition for an analysis outcome; rather, there is a range of possible decompositions that derives from the evidence theory structure (i.e., beliefs and plausibilities) assigned to the individual uncertain analysis inputs. Thus, the sensitivity measure for a particular analysis input will not be a unique variance contribution but rather a range of possible variance decompositions. More specifically, the variance decompositions associated with a particular analysis input will have an evidence theory structure that derives from the collective evidence theory structure assumed for all the inputs and the properties of the model(s) involved in the analysis. There will also be a correlation structure involving the variance decompositions associated with the individual analysis inputs.

The presented analysis strategy is to use a sampling procedure such as Latin hypercube sampling[18, 19] to develop a mapping between analysis inputs and analysis results. This mapping is then reused many times to develop the multiple possible variance decompositions and associated evidence theory structure (i.e., beliefs and plausibilities) for each analysis input. The result is a sensitivity analysis that uses an evidence theory structure to display the implications of the evidence theory representations of the uncertainty in each analysis input. However, the computational implementation of the analysis is based on making efficient use of ideas and procedures originally developed for use in analyses based on a probabilistic representation of uncertainty.

The analysis procedure is illustrated with a study of a hypothetical weak link/strong link system of the type used to assure the inoperability of high consequence systems under accident conditions.

References

1. Saltelli, A., K. Chan, and E.M. Scott, eds. *Sensitivity Analysis*. 2000, Wiley: New York.

2. Parry, G.W. and P.W. Winter, *Characterization and Evaluation of Uncertainty in Probabilistic Risk Analysis.* Nuclear Safety, 1981. **22**(1): p. 28-42.
3. Apostolakis, G., *The Concept of Probability in Safety Assessments of Technological Systems.* Science, 1990. **250**(4986): p. 1359-1364.
4. Helton, J.C., *Uncertainty and Sensitivity Analysis in the Presence of Stochastic and Subjective Uncertainty.* Journal of Statistical Computation and Simulation, 1997. **57**(1-4): p. 3-76.
5. Turányi, T., *Sensitivity Analysis of Complex Kinetic Systems. Tools and Applications.* Journal of Mathematical Chemistry, 1990. **5**(3): p. 203-248.
6. Cacuci, D.G., *Sensitivity and Uncertainty Analysis, Vol.1: Theory*. 2003, Boca Raton, FL: CRC Press.
7. Cukier, R.I., H.B. Levine, and K.E. Shuler, *Nonlinear Sensitivity Analysis of Multiparameter Model Systems.* Journal of Computational Physics, 1978. **26**(1): p. 1-42.
8. Sobol', I.M., *Sensitivity Estimates for Nonlinear Mathematical Models.* Mathematical Modeling & Computational Experiment, 1993. **1**(4): p. 407-414.
9. Rabitz, H., et al., *Efficient Input-Output Model Representations.* Computer Physics Communications, 1999. **117**(1-2): p. 11-20.
10. Saltelli, A., S. Tarantola, and K.P.-S. Chan, *A Quantitative Model-Independent Method for Global Sensitivity Analysis of Model Output.* Technometrics, 1999. **41**(1): p. 39-56.
11. McKay, M.D., J.D. Morrison, and S.C. Upton, *Evaluating Prediction Uncertainty in Simulation Models.* Computer Physics Communications, 1999. **117**(1-2): p. 44-51.
12. Iman, R.L. *Uncertainty and Sensitivity Analysis for Computer Modeling Applications*. in *Reliability Technology - 1992, The Winter Annual Meeting of the American Society of Mechanical Engineers, Anaheim, California, November 8-13, 1992*. 1992. Vol. 28. New York, NY: American Society of Mechanical Engineers, Aerospace Division.
13. Helton, J.C. and F.J. Davis, *Sampling-Based Methods for Uncertainty and Sensitivity Analysis*, in *Sensitivity Analysis*. 2000, Sandia National Laboratories: Albuquerque, NM.
14. Kleijnen, J.P.C. and J.C. Helton, *Statistical Analyses of Scatterplots to Identify Important Factors in Large-Scale Simulations, 1: Review and Comparison of Techniques.* Reliability Engineering and System Safety, 1999. **65**(2): p. 147-185.
15. Klir, G.J., *Is there more to uncertainty than some probability theorists might have us believe?* International Journal of General Systems, 1989. **15**: p. 347-378.
16. Klir, G.J., *On the Alleged Superiority of Probability Representation of Uncertainty.* IEEE Transactions on Fuzzy Systems, 1994. **2**: p. 27-31.
17. Klir, G.J. and M.J. Wierman, *Uncertainty-Based Information*. 1999, Physica-Verlag: New York, NY.
18. McKay, M.D., R.J. Beckman, and W.J. Conover, *A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code.* Technometrics, 1979. **21**(2): p. 239-245.
19. Helton, J.C. and F.J. Davis, *Latin Hypercube Sampling and the Propagation of Uncertainty in Analyses of Complex Systems.* Reliability Engineering and System Safety, 2003. **81**(1): p. 23-69.

[1]Corresponding Author
J.C. Helton
Dept 6849, MS 0779

Sandia National Laboratories
Albuquerque, NM 87185-0779
Ph: 505-284-4808
Fax: 505-844-2348
E-mail: jchelto@sandia.gov